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UCH O‘LCHAMLI QO‘ZG‘ALISHGA EGA FRIDRIXS MODELINING REZOLVENTASINI QURISH

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Annotatsiya. *Ushbu ishda uch o‘lchamli qo‘zg‘alishga ega Fridrixs modeli* $L_2[-\pi;\pi]$ *kompleks Hilbert fazosidagi chiziqli, chegaralangan va o‘z-o‘ziga qo‘shma operator sifatida qaralgan. O‘rganilayotgan Fridrixs modeliga mos rezolventa operatori Kramer usulidan foydalanib qurilgan.*

Kalit so‘zlar: *parametr funksiya, Fridrixs modeli, qo‘zg‘alish operatori, Kramer qoidasi, rezolventa operatori.*

Ushbu ishda $L_2[-\pi;\pi]$ kompleks Hilbert fazosida chiziqli, chegaralangan va o‘z-o‘ziga qo‘shma bo‘lgan hamda kvant mexanikasi, statistik mexanika va gidrodinamikaning ko‘plab dolzarb masalalari uchraydigan uch o‘lchamli qo‘zg‘alishga ega Fridrixs modelining rezolventa operatori qurilgan. Ta’kidlash joizki, tadqiq qilingan Fridrixs modelini panjaradagi ikki zarrachali sistemaga mos Hamiltonian sifatida ham qarash mumkin [1-5].

$L_2[-\pi;\pi]$ kompleks Hilbert fazosida

$$H = H_0 - V_1 - V_2 - V_3 \tag{1}$$

ko‘rinishdagi operatorni qaraymiz. Bu yerda H_0 operator $u(\cdot)$ funksiyaga ko‘paytirish operatori bo‘lib,

$$(H_0 f)(x) = u(x)f(x), \quad f \in L_2[-\pi;\pi],$$

V_α operatorlar esa qo‘zg‘alish operatorlari (integral operatorlar) bo‘lib,

$$(V_\alpha f)(x) = v_\alpha(x) \int_{-\pi}^{\pi} v_\alpha(t) f(t) dt, \quad \alpha = 1,2,3, \qquad f \in L_2[-\pi; \pi]$$

ko‘rinishda aniqlangan. Operatorning $u(\cdot)$, $v_1(\cdot)$, $v_2(\cdot)$ hamda $v_3(\cdot)$ parameter funksiyalar $[-\pi; \pi]$ kesmada aniqlangan haqiqiy qiymatli uzluksiz funksiyalar.

Parametr funksiyalarga qo‘yilgan bunday shartlarda (1) tenglik bilan aniqlangan H operator $L_2[-\pi; \pi]$ Hilbert fazosidagi chiziqli, chegaralangan va o‘z-o‘ziga qo‘shma operator bo‘ladi.

Odatda (1) tenglik yordamida aniqlangan operatorga Fridrixs modeli deb ataladi. $v_1(\cdot)$, $v_2(\cdot)$ va $v_3(\cdot)$ parametr funksiyalar chiziqli bog‘lanmagan bo‘lsin deb faraz qilinadi va bu holda $V_1 + V_2 + V_3$ uch o‘lchamli operator bo‘ladi.

Chekli o‘lchamli qo‘zg‘alishlarda muhim spektrning o‘zgarmasligi haqidagi Veyl teoremasiga ko‘ra H operatorning muhim spektri H_0 operatorning muhim spektri bilan ustma-ust tushadi. H_0 operator $u(\cdot)$ uzluksiz funksiyaga ko‘paytirish operatori bo‘lganligi bois, faqat sof muhim spektrga ega hamda

$$\sigma_{\text{ess}}(H) = \sigma(H_0) = \sigma_{\text{ess}}(H_0) = [m; M].$$

Bu yerda m va M sonlari

$$m := \min_{x \in [-\pi; \pi]} u(x), \qquad M := \min_{x \in [-\pi; \pi]} u(x)$$

tengliklar yordamida aniqlanadi.

H operatorning diskret spekri va rezolventasini aniqlash maqsadida $\mathbb{C} \backslash [m; M]$ sohada regulyar bo‘lgan

$$I_{\alpha\beta}(z) := \int_{-\pi}^{\pi} \frac{v_\alpha(t)\, v_\beta(t)}{u(t) - z} dt, \quad \alpha, \beta = 1,2,3$$

$$\Delta(z) := \begin{vmatrix} 1 - I_{11}(z) & -I_{12}(z) & -I_{13}(z) \\ -I_{21}(z) & 1 - I_{22}(z) & -I_{23}(z) \\ -I_{31}(z) & -I_{32}(z) & 1 - I_{33}(z) \end{vmatrix}$$

funksiyalarni hamda har bir $g \in L_2[-\pi; \pi]$ funksiya uchun

$$K_\alpha(g; z) = \int_{-\pi}^{\pi} \frac{v_\alpha(t)\, g(t)}{u(t) - z} dt, \quad \alpha = 1,2,3$$

funksiyani kiritamiz.

O‘rganilayotgan H operatorga mos rezolventa operatorini qurish uchun

$$\Delta_{k_1}(g;z):=\begin{vmatrix} K_1(g;z) & -I_{12}(z) & -I_{13}(z) \\ K_2(g;z) & 1-\ I_{22}(z) & -I_{23}(z) \\ K_3(g;z) & -I_{32}(z) & 1-\ I_{33}(z) \end{vmatrix};$$

$$\Delta_{k_2}(g;z):=\begin{vmatrix} 1-\ I_{11}(z) & K_1(g;z) & -I_{13}(z) \\ -\ I_{21}(z) & K_2(g;z) & -I_{23}(z) \\ -\ I_{31}(z) & K_3(g;z) & 1-\ I_{33}(z) \end{vmatrix};$$

$$\Delta_{k_3}(g;z):=\begin{vmatrix} 1-\ I_{11}(z) & -I_{12}(z) & K_1(g;z) \\ -\ I_{21}(z) & 1-\ I_{22}(z) & K_2(g;z) \\ -\ I_{31}(z) & -I_{32}(z) & K_3(g;z) \end{vmatrix}$$

determinantlarni tuzamiz. Yuqoridagi belgilashlardan va Kramer usulidan foydalanib H operatorga mos $R_z(H)$ rezolventa operatorini quyidagi

$$(R_z(H)g)(x)=\frac{\Delta_{k_1}(g;z)}{\Delta(z)}\cdot\frac{v_1(x)}{u(x)-z}+\frac{\Delta_{k_2}(g;z)}{\Delta(z)}\cdot\frac{v_2(x)}{u(x)-z}+$$
$$\frac{\Delta_{k_3}(g;z)}{\Delta(z)}\cdot\frac{v_3(x)}{u(x)-z}+\frac{g(x)}{u(x)-z}$$

tenglik orqali aniqlaymiz. Bunda $z\notin\sigma(H)$ bo‘lganligi bois $\Delta(z)\neq 0$ munosabat bajariladi.

Foydalanilgan adabiyotlar ro‘yxati.

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