DOIRAVIY CHEGARALANGAN UCH JISM MASALASI DOIRASIDA ORALIQ TORTISH QISMLARI UCHUN ANALITIK YECHIMLAR

Ruzmatov M.I., f-m.f.f.d. (PhD), dotsent

O'zbekiston milliy universiteti "Mexanika va matematik modellashtirish" kafedrasi dotsenti

Maxmudov X.X.

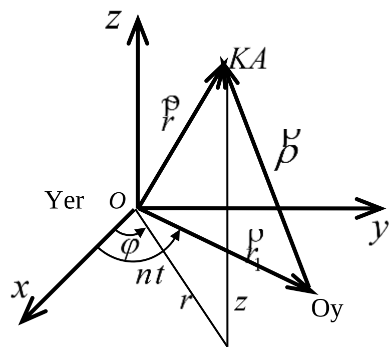
O'zbekiston milliy universiteti "Mexanika va matematik modellashtirish" kafedrasi magistranti

O'zgaruvchan massali nuqtani (kosmik apparatning (KA) massalar markazi) massalari M_1 va M_2 $(M << M_2 < M_1)$ bo'lgan ikkita tortish markazlari gravitatsion maydonidagi harakatini ko'rib chiqamiz. Qulaylik uchun bu markazlarni mos ravishda Yer va Oy deb nomlaymiz. Bunda massasi M bo'lgan nuqta tortish markazlarining harakatiga ta'sir ko'rsatmaydi hamda Oy Yer atrofida doiraviy orbita bo'ylab harakat qiladi deb qaraymiz.

Nuqtaning harakat differensial tenglamalari ushbu ko'rinishga ega [1,2]

$$\frac{d^2\vec{r}}{dt^2} = \frac{cm}{M}\vec{e} - \frac{\mu_1}{\tilde{r}^2}\frac{\vec{r}}{\tilde{r}} - \frac{\mu_2}{\rho^2}\frac{\left(\vec{r}-\vec{r}_1\right)}{\rho} - \frac{\mu_2}{a^2}\frac{\vec{r}_1}{a}. \qquad (1)$$

buyerda $\vec{r}$, $\vec{r}_1$ – mos ravishda nuqta (KA) va Oyning geosentrik radius-vektorlari, $r_1 = a$ – Er va Oy markazlari orasidagi o'rtacha masofa (1-rasm), $\tilde{r}^2 = r^2 + z^2$, $\rho^2 = r^2 + z^2 + a^2 - 2ar\cos\alpha$, $\alpha = nt - \varphi$, m – massa sarfi $(0 \le m(t) \le \tilde{m})$; $\vec{e}$ – tortish kuchi yo'nalishining birlik vektori; c yonish maxsulotlarining nisbiy tezligi miqdorini o'zgarmas deb hisoblaymiz, μ_1, μ_2 – mos ravishda Yer va Oyning gravitatsion parametrlari.



1-rasm Geotsentrik va silindrik koordinatalar sistemasi

(1) tenglamani geosentrik silindrik koordinatalar sistemasiga proyeksiyalab, harakat tenglamalarining birinchi guruhini olamiz [2]:

$$\dot v_1 = \frac{cm}{M}\lambda_1 - \frac{\mu_1 r}{\tilde r^3} - \frac{\mu_2}{\rho^3}(r - a\cos\alpha) - \frac{\mu_2}{a^2}\cos\alpha + \frac{v_2^2}{r},$$

$$\dot v_2 = \frac{cm}{M}\lambda_2 + \frac{\mu_2}{\rho^3}a\sin\alpha - \frac{\mu_2}{a^2}\sin\alpha - \frac{v_1 v_2}{r},$$

$$\dot v_3 = \frac{cm}{M}\lambda_3 - \frac{\mu_1 z}{\tilde r^3} - \frac{\mu_2 z}{\rho^3}, \qquad (2)$$

$$\dot r = v_1;\ \dot\varphi = \frac{v_2}{r};\ \dot z = v_3;\ \dot M = -m$$

Xarakteristik tezlikni minimallash haqidagi masalada gamiltonian quyidagicha kiritiladi [3,5]:

$$H = \lambda_1\left(\frac{cm}{M}\lambda_1 - \frac{\mu_1 r}{\tilde r^3} - \frac{\mu_2}{\rho^3}(r - a\cos\alpha) - \frac{\mu_2}{a^2}\cos\alpha + \frac{v_2^2}{r}\right) +$$

$$+\lambda_2\left(\frac{cm}{M}\lambda_2 + \frac{\mu_2}{\rho^3}a\sin\alpha - \frac{\mu_2}{a^2}\sin\alpha - \frac{v_1 v_2}{r}\right) + \lambda_3\left(\frac{cm}{M}\lambda_3 - \frac{\mu_1 z}{\tilde r^3} - \frac{\mu_2 z}{\rho^3}\right) + \qquad (3)$$

$$+\lambda_4 v_1 + \lambda_5\frac{v_2}{r} + \lambda_6 v_6 - \lambda_7 m.$$

Bu yerda v_1, v_2, v_3 silindrik sistemada $\vec v$ nuqta tezligini tashkil etuvchilari; $\lambda_i\left(i = \overline{1,7}\right)$ – $v_1, v_2, v_3, r, \varphi, z, M$ koordinatalarga qo‘shma ko‘paytuvchilar; $\vec\lambda = \vec\lambda(\lambda_1, \lambda_2, \lambda_3)$ - bazis-vektor, $|\vec\lambda| = 1$, $\vec e = \vec\lambda$; α – Oy orbitasi tekisligida KA va Oy orasidagi burchak.

Harakat tenglamalarining ikkinchi guruhi ushbu ko‘rinishga ega

$$\dot{\lambda}_1 = \lambda_2 \frac{v_2}{r} - \lambda_4$$

$$\dot{\lambda}_2 = \frac{1}{r}\left(\lambda_2 v_1 - 2\lambda_1 v_2 - \lambda_5\right)$$

$$\dot{\lambda}_3 = -\lambda_6$$

$$\begin{aligned}\dot{\lambda}_4 = {} & \lambda_1\left(\frac{\mu_1}{\tilde{r}^3} + \frac{\mu_2}{\rho^3} + \frac{v_2^2}{r^2} - \frac{3\mu_1 r^2}{\tilde{r}^5} - \frac{3\mu_2}{\rho^5}(r - a\cos\alpha)^2\right) + \\ & + \lambda_2\left(\frac{3\mu_2 a}{\rho^5}\sin\alpha\,(r - a\cos\alpha) - \frac{v_1 v_2}{r^2}\right) - \lambda_3\left(\frac{3\mu_1 r z}{\tilde{r}^5} + \frac{3\mu_2 z}{\rho^5}(r - a\cos\alpha)\right) + \lambda_5 \frac{v_2}{r^2}\end{aligned} \qquad (4)$$

$$\begin{aligned}\dot{\lambda}_5 = {} & \lambda_1 \mu_2 \sin\alpha\left(\frac{1}{a^2} - \frac{a}{\rho^3} + \frac{3ar}{\rho^5}(r - a\cos\alpha)\right) + \\ & + \lambda_2 \mu_2\left(\frac{a}{\rho^3}\cos\alpha - \frac{\cos\alpha}{a^2} - \frac{3a^2 r}{\rho^5}\sin^2\alpha\right) - \lambda_3 \cdot 3\mu_2 \frac{zar}{\rho^5}\sin\alpha\end{aligned}$$

$$\begin{aligned}\dot{\lambda}_6 = {} & -3\lambda_1 z\left(\frac{\mu_1 r}{\tilde{r}^5} + \frac{\mu_2}{\rho^5}(r - a\cos\alpha)\right) + \lambda_2 \frac{3\mu_2 a z}{\rho^5}\sin\alpha + \\ & + \lambda_3\left(\frac{\mu_1}{\tilde{r}^3} + \frac{\mu_2}{\rho^3} - \frac{3\mu_1 z^2}{\tilde{r}^5} - \frac{3\mu_2 z^2}{\rho^5}\right),\end{aligned}$$

$$\dot{\lambda}_7 = \frac{cm}{M^2}.$$

Variatsion masalaning differensial tenglamalari uchun oraliq tortish qismlarida faqat ikkita integral mavjud. Ular umumiy yechimni aniqlash uchun yetarli emas, shuning uchun xususiy integrallar va xususiy yechimlarni topish qiziqish uyg‘otadi. Ushbu ishda xususiy yechimlarni aniqlash uchun Dokshevich usulidan foydalanamiz.

Ushbu

$$F\left(v_1, v_2, \lambda_1, \lambda_2, \lambda_4, \lambda_5\right) = const, \qquad (5)$$

ko‘rinishdagi xususiy integralni qaraymiz [2, 4].

F funksiyadan vaqt bo‘yicha to‘liq hosila variatsion masala differensial tenglamalariga ko‘ra aynan nolga teng bo‘ladi. (5) integralda qatnashmaydigan o‘zgaruvchilar bu shart bilan bog‘lanmagani uchun ixtiyoriy bo‘lishi mumkin, u holda F funksiyadan vaqt bo‘yicha olingan to‘liq hosila ifodasida shu o‘zgaruvchilarning oldidagi koeffitsientlar nolga teng bo‘lishi kerak. Natijada xususiy hosilalarga nisbatan bir jinsli chiziqli algebraik tenglamalar sistemasi olinadi. Bu sistema noldan farqli yechimga ega bo‘lishi uchun uning bosh determinanti nolga teng bo‘lishi kerak

$$\begin{vmatrix} \lambda_1 & \lambda_2 & 0 & 0 & 0 & 0 \\ r & 0 & 0 & 0 & -\lambda_1 & 0 \\ -b & a\sin\alpha & 0 & 0 & \lambda_1 & -aD \\ 0 & 0 & 0 & 0 & -Cb & Car\sin\alpha \\ v_2^2 & -v_1 v_2 & \lambda_2 v_2 & A & 0 & 0 \\ \frac{\mu_2}{a^2}\cos\alpha & \frac{\mu_2}{a^2}\sin\alpha & \lambda_4 & 0 & B & -\frac{\mu_2}{a^2}D \end{vmatrix} = 0$$

yoki

$$\lambda_4(2\lambda_1 v_2 - \lambda_2 v_1 + \lambda_5)(\lambda_1 r - \lambda_1 a\cos\alpha - \lambda_2 a\sin\alpha)(\lambda_2\cos\alpha - \\ - \lambda_1\sin\alpha)(\lambda_1 a\sin\alpha - \lambda_2 a\cos\alpha + \lambda_2 r) = 0. \quad (6)$$

Har bir qavsdagi ifodalarni nolga tenglab, turli xususiyyechimlarga olib keluvchi invariant munosabatlarni olish mumkin. (6) shart asosida $z = 0$ ekanligi olingan, yani KA Oy orbitasi tekisligida harakat qiladi. Shunday ekan $v_3 = 0,\ \lambda_3 = 0$.

Xususiy yechimlarni aniqlash uchun (6) da quyidagi ifoda nolga teng bo‘lsin deylik

$$\lambda_1 a\sin\alpha - \lambda_2 a\cos\alpha + \lambda_2 r = 0 \quad (7)$$

Masalaning qo‘yilishidagi $\lambda_1^2 + \lambda_2^2 = 1$ tenglikdan $\dot{\lambda}_2 = 0$, $\lambda_1\dot{\lambda}_1 = 0$. Ikkita variant olish mumkun; 1) $\lambda_2 = 0$, $\lambda_1 = \pm 1$; 2) $\lambda_1 = 0$, $\lambda_2 = \pm 1$. birinchi variantdan ko‘rinib turibdiki (7) tenglik $\lambda_1 a\sin\alpha = 0$. buyerda $a \neq 0$ va

$$\sin\alpha = 0; \quad \cos\alpha = \pm 1, \quad (8)$$

$\lambda_2 = 0$ va $\sin\alpha = 0$ ekanligidan foydalanib masalaning qo‘yilishida berilgan $\dot{v}_2$ ni topadigan bo‘lsak $\dot{v}_2 = -\frac{v_1 v_2}{r}$ bundan $\frac{dv_2}{v_2} = -\frac{dr}{r}$. Ushbu tenglikni integratsiyalab

$$v_2 = \frac{v_{20}}{r} r_0 \quad (9)$$

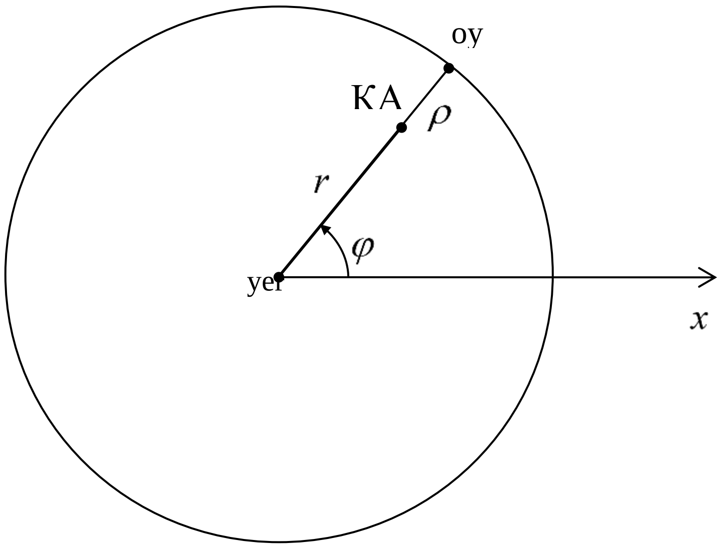
Tezlikning ko‘ndalang komponenti v_2 qo‘zg‘almas markazgacha bo‘lgan masofa r ga teskari proportsional ravishda o‘zgaradi.
Agar $v_{20} > 0$, bo‘lsa $v_2 > 0$; agar $v_{20} < 0$, bo‘lsa $v_2 < 0$. Boshqa tomondan $v_2 = r\dot{\varphi}$ (9) bilan solishtirsak

$$\dot{\varphi} = \frac{v_{20} r_0}{r^2} \qquad (10)$$

Nuqtaning burchak tezligi r^2 ga teskari proporsionaldir. (8) dan kelib chiqadiki $\alpha = 0$, yoki $\alpha = \pi$.

1) agar bizda $\alpha = 0$ bo'lganda

$$nt - \varphi = 0, \ \dot{\varphi} = n \qquad (11)$$



2-rasm. KA joylashuvi $\alpha = 0$ bo‘lganda

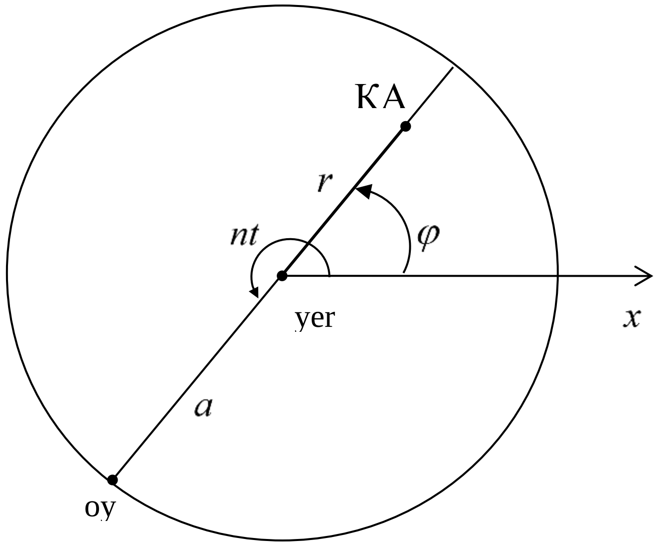
Kosmik kema Oyning markazidan o‘tadigan Oy orbitasining radiusida joylashgan (2-rasm). Kosmik kema va Oyning burchak tezligi bir-biriga to‘g‘ri keladi, $\rho = a - r$

2)agar bizda $\alpha = \pi$ bo‘lsa

$$nt - \varphi = \pi, \ \dot{\varphi} = n. \qquad (12)$$

Kosmik kema Oydan π burchagi bo‘yicha orqada qoladi (yoki oldinda boradi), ya’ni Oyning markazidan o‘tadigan Oy orbitasi radiusining kengaytmasida joylashgan (3-rasm). $\rho = a + r$. Kosmik kema va Oyning burchak tezliklari bir-

biriga mos keladi.



3-rasm. KA joylashuvi $\alpha = \pi$ bo'lganda

(10),(11),(12) larga ko'ra bizda $\frac{v_{20} r_0}{r^2} = n$, buyerdan $r^2 = \frac{v_{20} r_0}{n}$. Shunday qilib nuqta (KA) radiusi $r_0 = \frac{v_{20}}{n}$ bo'lgan aylana bo'ylab bir tekis ($v_2 = r_0 n$) harakat qiladi. v_2 ning kattaligi dastlabki masofa r_0 ga bog'liq. Radial tezlik komponenti $v_1 = \dot r = 0$

$\dot v_1 = \frac{\partial H}{\partial \lambda_1}$ tenglamadan massaning o'zgarishini topamiz, bu bizning holatimizda quyidagi ko'rinishni oladi.

$$0 = \frac{cm}{M}\lambda_1 - \frac{\mu_1}{r^2} - \frac{\mu_2}{\rho^3}(r - a\cos\alpha) - \frac{\mu_2}{a^2}\cos\alpha + n^2 r \qquad (13)$$

Bu yerda ham ikki variantni ko'rib chiqamiz .

________1). Agar bizda $\alpha = 0$, $\cos\alpha = 1$, $\rho = a - r$ (2-rasm) bo'lsa

$$\frac{cm}{M}\lambda_1 = \frac{\mu_1}{r^2} - \frac{\mu_2}{\rho^2} + \frac{\mu_2}{a^2} - n^2 r \qquad (14)$$

(14) da o'ng tomonni $A(\mu_1, \mu_2, r)$ bilan belgilaymiz. λ_1 va A belgilari bir xil. Bundan tashqari

$$\mu_1 = b\mu_2, \quad b > 1, \quad n^2 = \frac{\mu_1 + \mu_2}{a^3}. \tag{15}$$

Demak $A = \frac{b\mu_2}{r^2} - \frac{\mu_2}{\rho^2} + \frac{\mu_2}{a^2} - \frac{b\mu_2 r}{a^3} - \frac{\mu_2 r}{a^3} = \frac{\mu_2}{a^3}(b\frac{a^3 - r^3}{r^2} + \frac{\rho^3 - a^3}{\rho^2})$

Agar $b\frac{a^3 - r^3}{r^2} > \frac{a^3 - \rho^3}{\rho^2}$, bo‘lsa $A > 0$ va (14) da $\lambda_1 > 0$, ya’ni $\lambda_1 = 1$ (16)

Agar $b\frac{a^3 - r^3}{r^2} < \frac{a^3 - \rho^3}{\rho^2}$, bo‘lsa $A < 0$ va (14) da $\lambda_1 < 0$ ya’ni $\lambda_1 = -1$ (17)

Shunday qilib λ_1 va A belgilari $r_0\,(\rho = a - r)$ nuqtasining boshlang‘ich holatiga va μ_1, μ_2 parametrlarning nisbatiga bog‘liq.

(14) dan bizga ma’lum:

$$-c\frac{dM}{M}\lambda_1 = Adt;$$

$$\ln M = -\frac{A}{c\lambda_1}t + \ln M_0$$

$$M = M_0 e^{\frac{-A}{c\lambda_1}t}.$$

$\frac{A}{\lambda_1} > 0$ bo‘lgani uchun $M(t)$ ning massasi ekspotensial qonunga ko‘ra kamayadi

Ikkita holatni qarasak. (16) va (17) dan shuni ko‘ramizki $a > r$ bo‘lgani uchun

$b > \frac{a^3 - \rho^3}{a^3 - r^3}.\frac{r^2}{\rho^2} > 1$, dan $\lambda_1 > 0$, va agar $1 < b < \frac{a^3 - \rho^3}{a^3 - r^3}.\frac{r^2}{\rho^2}$, dan $\lambda_1 < 0$.

Ikkala holatda ham $(\frac{a^3 - \rho^3}{a^3 - r^3})\frac{r^2}{\rho^2} > 1$ ekanligi ma’lum.

$$\frac{(a-\rho)(a^2 + a\rho + \rho^2)}{(a-r)(a^2 + ar + r^2)}\frac{r^2}{\rho^2} > 1$$

$$\frac{r^3(a^2 + a\rho + \rho^2)}{\rho^3(a^2 + ar + r^2)} > 1 \tag{18}$$

O‘zgartirishlardan so‘ng (18) tengsizlik

$$\frac{\frac{a^2}{\rho^3}+\frac{a\rho}{\rho^3}+\frac{\rho^2}{\rho^3}}{\frac{a^2}{r^3}+\frac{ar}{r^3}+\frac{r^2}{r^3}}>1 \quad \text{dan =>} \quad \frac{\frac{a^2}{\rho^3}+\frac{a}{\rho^2}+\frac{1}{\rho}}{\frac{a^2}{r^3}+\frac{a}{r^2}+\frac{1}{r}}>1 \qquad (19)$$

Shundan so‘ng (19) tengsizlikni qanoatlantirish uchun quyidagilar yetarli.

$$\rho<r, \quad r>\frac{a}{2} \qquad (20)$$

________2) Agar bizda $\alpha=\pi$, $\cos\alpha=-1$, $\rho=a+r$ (3-rasm) bo‘lsa

$$\frac{cm}{M}\lambda_1=\frac{\mu_1}{r^2}+\frac{\mu_2}{\rho^3}(a+r)-\frac{\mu_2}{a^2}-n^2r \qquad (21)$$

(21) da o‘ng tomonni $B(\mu_1,\mu_2,r)$ bilan belgilaymiz. λ_1 va B belgilari bir xil. (15) va $a>r$, $a<\rho$larni hisobga olib.

$$B=\frac{b\mu_2}{r^2}+\frac{\mu_2}{\rho^2}-\frac{\mu_2}{a^2}-\frac{b\mu_2 r}{a^3}-\frac{\mu_2 r}{a^3}=\frac{\mu_2}{a^3}(b\frac{a^3-r^3}{r^2}-\frac{\rho^3-a^3}{\rho^2})$$

Agar $b>\frac{\rho^3-a^3}{a^3-r^3}.\frac{r^2}{\rho^2}>1$, bo‘lsa $B>0$ va (21) da $\lambda_1>0$, ya’ni $\lambda_1=1$ (22)

Agar $1<b<\frac{\rho^3-a^3}{a^3-r^3}.\frac{r^2}{\rho^2}$, bo‘lsa $B<0$ va (21) da $\lambda_1<0$ ya’ni $\lambda_1=-1$ (23)

Ikkala holatda ham $(\frac{\rho^3-r^3}{a^3-r^3})\frac{r^2}{\rho^2}>1$ ekanligi ma’lum

$$\frac{(\rho-a)(a^2+a\rho+\rho^2)}{(a-r)(a^2+ar+r^2)}\frac{r^2}{\rho^2}>1$$

buyerda $(a-r)=(2a-\rho)$

$$\frac{r^3(a^2+a\rho+\rho^2)}{\rho^2(2a-\rho)(a^2+ar+r^2)}>1 \qquad (24)$$

Shunday qilib (24) tengsizligimizga quyidagicha o‘zgartirishlarni amalga oshirsak

$$\frac{\frac{a^2}{\rho^2(2a-\rho)}+\frac{a\rho}{\rho^2(2a-\rho)}+\frac{\rho^2}{\rho^2(2a-\rho)}}{\frac{a^2}{r^3}+\frac{ar}{r^3}+\frac{r^2}{r^3}}>1 .$$

$$\frac{\frac{a^2}{\rho^2(2a-\rho)}+\frac{a}{\rho(2a-\rho)}+\frac{1}{2a-\rho}}{\frac{a^2}{r^3}+\frac{a}{r^2}+\frac{1}{r}}>1 \qquad (25)$$

Bu tengsizlikni qanoatlantirishimiz uchun $2a-\rho<r$, $r>\frac{a}{2}$ lar yetarli, ya'ni nuqta(KA) radiusi $\frac{a}{2}$ dan katta bo'lishi kerak.

Shunday qilib λ_1 va B belgilari $r_0>\frac{a}{2}$ nuqtaning dastlabki holatiga va μ_1,μ_2 tortishish parametrlarining nisbatiga bog'liq.

(21)dan bizga ma'lum:

$$-c\frac{dM}{M}\lambda_1=Bdt;$$

$$\ln M=-\frac{B}{c\lambda_1}t+\ln M_0$$

$$M=M_0e^{\frac{-B}{c\lambda_1}t}$$

$\frac{B}{\lambda_1}>0$ bo'lgani uchun $M(t)$ ning massasi ekspotensial qonunga ko'ra kamayadi.

Shunday qilib, ushbu variantda nuqta(KA) Oy orbitasi tekisligida $a>r_0>\frac{a}{2}$ radiusli aylana yoy bo'ylab bir xilda harakat qiladi, Oy va nuqtaning(KA) burchak tezliklari mos keladi. Mos kelmaslik burchagi 0 yoki π. Nuqtaning massasi ekspotensial qonunga muvofiq kamayadi. Tortish kuchi radialdir, uning yo'nalishi nuqtaning dastlabki holatiga va tortish markazlari tortishish parametrlarining nisbatiga bog'liq.

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