Normal subgroups of index 6 for the group representation of the Cayley tree

[1]G'aybullayev Dilshodbek Erkin òg'li, [2]Xalilov Akbar Zamirovich

[1]gaybullayevdilshodbek0099@gamil.com ,[2]akbarx1991@gmail.com

[1]Karshi state university, [2]Karshi state university

There are several thousand papers and books devoted to the theory of groups. But still there are unsolved problems, most of which arise in solving of problems of natural sciences as physics, biology etc. In particular, if configuration of physical system is located on a lattice (in our case on the graph of a group) then the configuration can be considered as a function defined on the lattice. There are many works devoted to several kind of partitions of groups (lattices) (see e.g. [1]-[3]).

Let G_k be a free product of $k+1$ cyclic groups of the second order with generators $a_1, a_2, ... a_{k+1}$, respectively and the group G has a finitely generators of the order two and r is a minimal number of such generators of the group G and without loss of generality we can take these generators are $b_1, b_2, ... b_r$. Let e_1 is an identity element of the group G. We define homomorphism from G_k onto G. Let $\Xi_n = \{A_1, A_2, ..., A_n\}$ be a partition of $N_k \setminus A_0$, $0 \le |A_0| \le k+1-n.$ Then we consider homomorphism $u_n : \{a_1, a_2, ..., a_{k+1}\} \to \{e_1, b_1 ..., b_m\}$ as

$$u_n(x) = \begin{cases} e_1, \ if \ \ x = a_i, i \in A_0 \\ b_j, if \ \ x = a_i, i \in A_j, \ j = \overline{1, n} \end{cases} \tag{1}$$

For $b \in G$ we denote $R_b[b_1, b_2, ..., b_m]$ is a representation of the word b by generators $b_1, b_2, ..., b_r, r \le m.$ Define the homomorphism $\gamma_n : G \to G$ by the formula

$$\gamma_n(x)=\begin{cases} e_1,\ if\ \ x=e_1 \\ b_i,\ if\ \ x=b_i, i=\overline{1,r} \\ R_{b_i}[b_1,...,b_r],\ if\ \ x=b_i, i\neq\overline{1,r} \end{cases} \qquad (2)$$

Theorem. For the group G_k following statement is hold

$$\{H \mid H \text{ is a normal subgroup of } G_k \text{ with} \mid G_k : H \mid = 8\} =$$
$$= \{H^{(4)}_{C_0C_1C_2}(D_4) \mid C_1, C_2 \text{ is a partition of } N_k \setminus C_0\} \cup \Re.$$

Reference

1. Ganikhodjaev, N.N., Rozikov, U.A., (1997), Description of periodic extreme Gibbs measures of some lattice model on the Cayley tree, Theor.Math.Phys. 111, pp. 480-486.

2. U.A., Rozikov, F.H., Haydarov., (2014), Normal subgroups of finite index for the group represantation of the Cayley tree, TWMS Jour.Pure.Appl.Math. 5, pp. 234-240.

3. U.A., Rozikov., (2013) Gibbs measures on a Cayley trees, World Sci. Pub, Singapore. Normal Subgroups of Index 8 in the Group Representation of the Cayley Tree