"LOGARIFMIK TENGLAMALAR"

Toshkent shahar Arxitektura-qurilish qoshidagi akademik litseyi
matematika fani o'qituvchisi
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Logarifmlar va ularning asosiy xossalari

Quyidagi misollarni ko`ramiz:

1. $2^x=4$ ni yechish uchun $2^x=2^2$ deb yozamiz va x=2 yechimni topamiz.

2. $2^x=5$ bo`lsin. o`ng tomondagi 5 ni asosi 2 bo`lgan daraja ko`rini-shida tasvirlash mushkul. Lekin bu tenglamaning haqiqiy ildizi mavjud-ligi bizga ma`lum. Bunday tenglamalarni yechish uchun logarifm tu-shunchasi kiritiladi.

Umuman olganda, $a^x=b$ $(a>0,\ a\neq 1,\ b>0)$ tenglamaning ildizi *a* asosga ko`ra *b* sonning logarifmi deyiladi.

**Ta`rif:** *b* sonning *a* asosga ko`ra logarifmi deb *b* sonni hosil qilish uchun *a* sonni ko`tarish kerak bo`ladigan daraja ko`rsatkichiga aytiladi va $log_a b$ kabi belgilanadi. $a^x=b$ tenglamani ($x=log_a b$ bo`lgani uchun)

$$a^{\log_a b}=b \qquad (1)$$

ko`rinishida yozish mumkin. (1) formula asosiy logarifmik ayniyat deyi-ladi, bu yerda

$a>0$ $a\neq 1$ va $b>0$

Misollar: 1) $log_2 16$ 2) $log_5 0{,}04$ ning qiymatini toping.

Yechish: 1) $16=2^4$ bo`lgani uchun, 16 ni hosil qilish uchun ikkini to`rtinchi darajaga ko`tarish kerak, demak $log_2 16=4$.

2) $0{,}04=\frac{4}{100}=\frac{1}{25}=5^{-2}$ ekanligi ma`lum. Shuning uchun *log₅0, 04=-2*

Misollar: 3. $\log_4 x=\frac{1}{2}$, 4) $\log_x 4=-\frac{3}{4}$ tenglamalarni qanoatlantiruv-chi x larni topamiz.

Yechish: Asosiy logarifmik ayniyatdan foydalanib:

$$3)\ x=4^{\frac{1}{2}}=2$$

4) $x^{\log_x 4}=4$, ya`ni $x^{-\frac{3}{4}}=4$, $x=4^{-\frac{4}{3}}=\frac{1}{\sqrt[3]{256}}$ larni topamiz.

Har qanday $a>0, b>0, a\neq 1, b\neq 1, x>0, y>0$ va haqiqiy istalgan n va m sonlar uchun quyidagi tengliklar bajariladi:

1) $\log_a 1=0$, 2) $\log_a a=1$,

3) $\log_a (xy)=\log_a x+\log_a y$,

4) $\log_a \frac{x}{y}=\log_a x-\log_a y$,

5) $\log_a x^n=n\log_a x$,

6) $\log_{a^m} x=\frac{1}{m}\log_a x$,

7) $\log_{a^m} x^n=\frac{n}{m}\log_a x$,

8) $\log_a x=\frac{\log_b x}{\log_b a}$,

9) $\log_a b=\frac{1}{\log_b a}$,

Bu tengliklar ko`rsatkichli funksiya xossalaridan kelib chiqadi. Bulardan ba`zilarini isbot qilamiz.

Logarifmik ayniyatdan foydalanib:

$x = a^{\log_a x}$, $y = a^{\log_a y}$ ni topamiz.

Bu tengliklarni hadlab ko`paytirsak yoki bo`lsak

$xy = a^{\log_a x} * a^{\log_a y} = a^{\log_a x + \log_a y}$,

$\frac{x}{y} = a^{\log_a x} : a^{\log_a y} = a^{\log_a x - \log_a y}$, hosil bo`ladi.

Bu tengliklardan logarifm ta`rifiga ko`ra 3) va 4) tengliklar kelib chiqadi.

$x = a^{\log_a x}$ ayniyatning ikkala tomonini n □ darajaga oshirsak, $x^n = a^{n\log_a x}$ hosil bo`lib, bundan $\log_a x^n = n\log_a x$ ni topamiz.

Bir asosli logarifmdan boshqa asosli logarifmga o`tish formulasi 8) ni xususiy holda 9) ni isbotlash uchun quyidagicha amal qilamiz:

$$\log_a x = b \Rightarrow x = a^b$$

Hosil bo`lgan $x=a^b$ ifodaning ikkala tomonidan $b$ asosga ko`ra logarifm topamiz:

$$\log_b x = \log_b a^b = b\log_b a \Rightarrow b = \frac{\log_b x}{\log_b a}$$

Chap tomonga b ning qiymatini qo`yib, 8) formulani hosil qilamiz. Agar bu formuladan $x=b$ desak, 9) formula hosil bo`ladi.

5-misol. Agar $\log_2 5 = a$ va $\log_2 3 = b$ bo`lsa, $\log_2 3000$ ni a va b orqali ifodalang?

Yechish: $\log_2 3000 = \log_2(3 \cdot 5^3 \cdot 2^3) = \log_2 3 + 3\log_2 5 + 3\log_2 2 = b + 3a + 3$

6-misol. Agar $\log_3 x = \log_3 7 + 2\log_3 5 - 3\log_3 2$ bo`lsa, x ni toping.

Yechish: $\log_3 x = \log_3 7 + \log_3 5^2 - \log_3 2^3 = \log_3 \frac{7 \cdot 5^2}{2^3} = \log_3 \frac{175}{8}$,

Bundan $x = \frac{175}{8} = 21{,}875$

12.2. O`nli va natural logarifmlar

**1-ta`rif.** Asosi *a=10* bo`lgan logarifmlar o`nli logarifmlar deyiladi va *lgx* orqali ifodalanadi, ya`ni $log_{10}x=lgx$

7-misol. $lg100=lg10^2=2$

$8: lg0,01=lg10^{-2}=-2$

**2-ta`rif.** Natural logarifm deb asosi *e* son bo`lgan logarifmga aytiladi va *lnx* bilan belgilanadi, ya`ni $log_e x=lnx$, *e* soni irratsional son bo`lib, *e=2,7182818284*… amalda *e≈2,7* deb qabul qilish mumkin.

O`nli va natural logarifmlar orasida

$\lg x = \frac{1}{\ln 10}\cdot \ln x \approx 0,434294\ln x$ va

$\ln x = \frac{1}{\lg e}\cdot \lg x \approx 2,302551\lg x$ bog`lanish mavjud. Amalda $\lg x \approx 0,4\ln x$ va $\ln x \approx 2,3\lg x$ tengliklardan foydalanish mumkin.

9-misol. *ln100*, lge^2 ni hisoblang.

Yechish: $\ln 100 \approx 2,3\cdot \lg 100 = 2,3\cdot 2 = 4,6.$
$\lg e^2 = 2\lg e \approx 2\cdot 0,4\ln e = 0,8.$

Mashqlar

Quyidagi logarifmlarni toping:

298) 1) $\log_3 9$; 2) $l\log_2 \frac{1}{16}$; 3) $\log_4 16$; 4) $\log_5 \frac{1}{25}$.

299) 1) $\log 3\sqrt[3]{3}$; 2) $\log_2 32^{-5}$; 3) $\log_7 7^0$; 4) $\log_3 27$.

300) 1) $\log_9 \frac{1}{81}$; 2) $\log_4 32$; 3) $\log_{\sqrt{2}} 8$; 4) $\log_{0,2} 125$.

Hisoblang:

301) 1) $2^{\log_2 13}$, 2) $3^{\log_3 9}$, 3) $\left(\frac{1}{3}\right)^{\log_{0,5} 3}$, 4) $\left(\frac{1}{4}\right)^{\log_{0,25} 3}$.

302) 1) $8^{\log_2 3}$, 2) $9^{\log_3 4}$, 3) $(0,25)^{\log_2 3}$, 4) $(0,04)^{\log_5 4}$.

303) 1) $\log_{\sqrt{2}} \sqrt[3]{2}$; 2) $\log_{\frac{1}{2}} \sqrt[3]{9}$; 3) $\log_3 \log_2 2^9$;

4) $\log_9 \lg 1000$; 5) $\log_4 \log_{32} 1024 - \log_{\frac{1}{2}} 4$;

6) $\log_8 5 + \log_8 40 - \log_8 15$;

7) $\dfrac{\log_2 12 - \frac{1}{2}\log_2 36}{\log_3 9 - \frac{1}{3}\log_3 9}$; 8) $\dfrac{6\log_7 2 - \log_7 64}{8\log_5 2 + \frac{2}{3}\log_5 27}$;

9) $\dfrac{2\log_5 3}{\log_{25} 27}$; 10) $\dfrac{\log_{27} 8}{\log_9 2}$.

304) 1) $\log_3 4$; 2) $\log_7 15$; 3) $\log_{0,8} 9$; 4) $\log_{1,3} 12$.

Berilgan logarifmlarni natural logarifmlar bilan almashtirib, mikro-kalkulatorda 0,01 aniqlik bilan hisoblang:

305) 1) $\log_2 5$; 2) $\log_5 4$; 3) $\log_7 25$; 4) $\log_{45} 9$.

306) 1) $\log_3 4$; 2) $\log_7 15$; 3) $\log_{0,8} 9$; 4) $\log_{1,3} 12$.

Javoblar: 298. 2) -4; 4) -2. 299. 2) -25; 4) 3. 300. 2) 3,5; 4) -3.

301. 2) 9; 4) 3. 302. 2) 16. 4) $\frac{1}{16}$; 303. 2) $-\frac{2}{3}$; 4) $\frac{1}{2}$.

6) $\log_8 \frac{40}{3}$; 8) 0; 10) 2.

12.3. Logarifmik funksiya va uning grafigi

Logarifmik funksiya deb, 2

$$y=log_a x$$

funksiyaga aytiladi. bu yerda $a>0,\ a\neq 1$. Funksiyaning ba`zi xossalarini ko`rib chiqamiz:

1) Funksiyaning aniqlanish sohasi $x>0$. Bu logarifmning ta`rifidan kelib chiqadi.

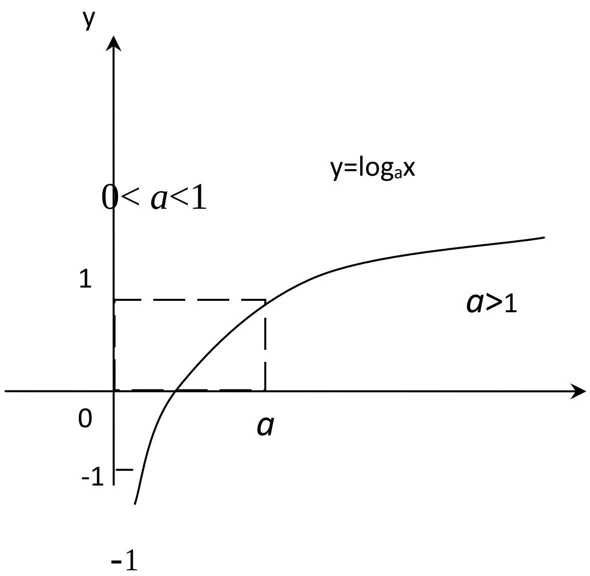
2) Logarifmik funksiyaning qiymatlar sohasi barcha haqiqiy sonlar-dan iborat. Haqiqatda, har qanday haqiqiy son b uchun shunday musbat x mavjudki, $log_a x=b$ bo`ladi, ya`ni $log_a x=b$ tenglama ildizga ega bo`ladi.

3) Barcha $x>0$ uchun agar $a>1$ bo`lsa logarifmik funksiya o`suvchi bo`ladi. Agar $0<a<1$ bo`lsa, kamayuvchi bo`ladi.

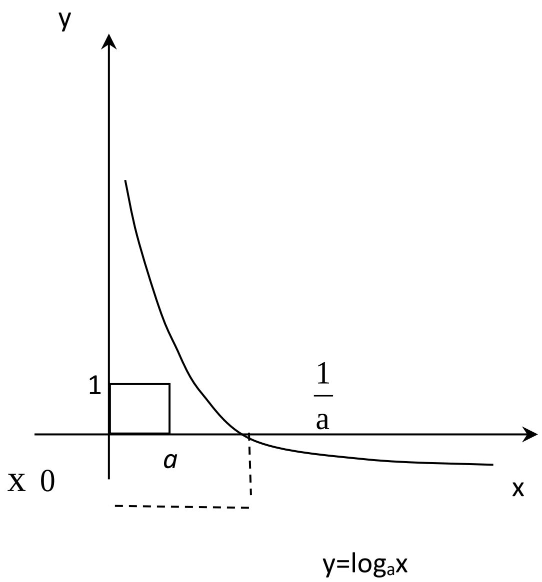
Haqiqatda $a>1$ bo`lganda $x_2>x_1$ uchun $\log_a x_2 > \log_a x_1$ bo`ladi va funksiya o`sadi. Agar $0<a<1$ bo`lsa, $a^{\log_a x_2} > a^{\log_a x_1}$ dan $\log_a x_2 > \log_a x_1$ kelib chiqadi. Bu funksiya kamayuvchiligini bildiradi.

4) $a>1$ bo`lganda, $y=log_a x$ funksiya $0<x<1$ uchun manfiy va $x>1$ uchun musbat qiymatlar qabul qiladi: $0<a<1$ bo`lganda, $0<x<1$ uchun funksiya musbat va $x>1$ uchun manfiy qiymatlar qabul qiladi. Bu xossa $y=log_a x$ funksiyaning o`suvchi ($a>1$) va kamayuvchi ($0<a<1$) ekanligi-dan kelib chiqadi. $x=1$ bo`lsa, $y=0$ grafik (1, 0) nuqtadan o`tadi.

5) Keltirilgan xossalardan foydalanib, funksiya grafigini yasaymiz. Ko`rinadiki grafik Oy o`qdan o`ngda joylashgan (47, 48 rasmlar).



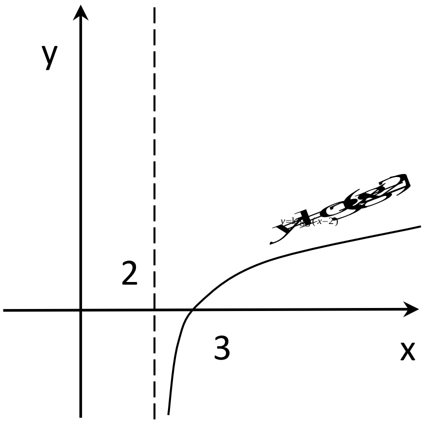
47-rasm.

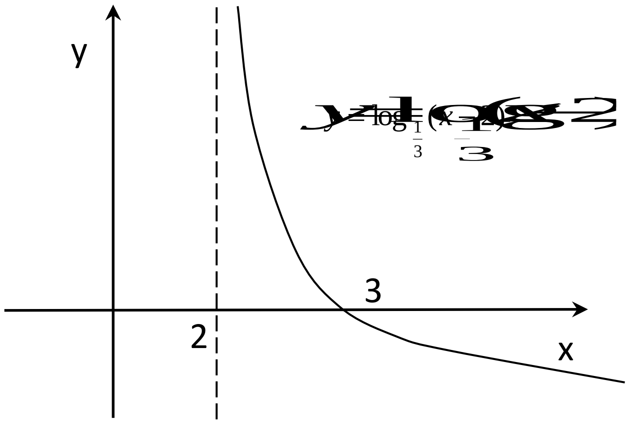


48-rasm.

Misollar. 1) $y=log_3(x\text{-}2)$, 2) $y=\log_{\frac{1}{3}}(x-2)$ funksiyaning grafik-larini yasang.

Yechish: Bu funksiyalarning grafiklari $y=\log_3 x$ va $y=\log_{\frac{1}{3}} x$ funksiyalarning grafiklarini Ox o`q bo`yicha o`ng tomonga ikki birlikka surishdan hosil bo`ladi (49 va 50- rasmlar).





3-misol. $y=\log_3|3-x|$ funksiyaning grafigini yasang.

Yechish: $x\in(-\infty,3)\cup(3,\infty)$ $x=3$ to`g`ri chiziqqa nisbatan grafik simmetrik joylashgan. Shuning uchun grafikni $x>3$ holat uchun yasab, uni $x=3$ to`g`ri chiziqqa nisbatan akslantirsak, $y=\log_3|3-x|$ funksiyaning grafigi hosil bo`ladi (51-rasm).