KO‘P O‘LCHOVLI MATEMATIKA VA FIZIKA TENGLAMALARI UCHUN NOKORREKT CHEGARAVIY MASALANI REGULARLASHTIRISH

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Annotatsiya: Ushbu maqolada biz tayin bir masala olib uni kvaziteskari usul bilan issiqlik tarqalish tenglamasiga qo‘yilgan teskari Koshi masalasini regulyarizatsiyasi bilan tanishamiz. Ayni vaqtda nokorrekt va teskari qo‘yilgan masalalar fani bo‘yicha regulyarizatsiyalash mavzusini uchun yaxshi yechim bo‘lib xizmat qiladi. Teskari qo‘yilgan masalalar qo‘yiladigan qo‘yiladigan shartlarni qanoatlantirmasligini hisobga olgan holda yechimga boshqacharoq yondashish kerak bo‘ladi

Kalit so‘zlar: kvaziteskari usul, Koshi masalasi, Furye koeffitsienti, issiqlik tarqalish tenglamasi,regulyarizatsiya.

$Q = (-1 < x < 1) \times (0, T)$ sohada quydagi tenglamani qaraymiz:

$$sign(x)u_t(x,t)=u_{tt}(x,t). \qquad (1)$$

(1) - chi tenglamani $Q(x\neq 0)$ da quydagi

a) boshlang’ich

$$u(x,0) = f(x), \quad -1\leq x \leq 1;$$

b) chegaraviy

$$U(-1,t)=u(1,t)=0, \ 0\leq t\leq T;$$

c) $u(-0,t)=u(+0,t)$, $u_z(-0,t)=u_z(+0,t)$, $0\leq t\leq T$.

shartlarni qanoatlantiruvchi yechimlarni toping, bu masala yefchimi mos fazoda yagona va shartli to‘g’ridir.

Faraz qilaylik, boshlang’ich berilganlar shundayki bu masalaning yechimi mavjud, uni quydagi ko‘rinishda ifodalash mumkin.

$$u(x,t) = \sum_{k=1}^{\infty} u_k^+(t)\varphi_k^+(x) + \sum_{k=1}^{\infty} u_k^-(t)\varphi_k^-(x) \ (2)$$

bu yerda

$$u_k^+ = exp(\lambda_k^+ t) f_k^+, u_k^- = exp(\lambda_k^- t) f_k^- ;$$

$$f_k^{\pm} = -\int_{-1}^{1} sign(x) f(x)\, \varphi_k^{\pm}(x)dx, \qquad k = 1,2,\ldots,$$

$$\varphi_k^{\pm} = \begin{cases} \mathrm{sh}\sqrt{\lambda_i^{\pm}}\sin\sqrt{\lambda_i^{\pm}}(1-x), \text{agarda } 0 \le x \le 1 \text{ bo'sa}; \\ \sin\sqrt{\lambda_i^{\pm}}\mathrm{sh}\sqrt{\lambda_i^{\pm}}(1-x), \text{agarda } -1 \le x \le 0 \text{ bo'sa}; \end{cases} \quad (3)$$

$$\pm\lambda_i^{\pm} = \mu_i^2, \ i = 1,2,\ldots,$$

μ lar esa

$$\mathrm{tg}\mu + \mathrm{th}\mu = 0$$

tenglamaning musat ildizlari.

$(u,v) = \int_{-1}^{1} uvdx$ bilan $L_i(-1,1)$ dagi skalar ko‘paytmani belgilasak, u holda $\|u\|^2=(u,u)$. (3) kelib chiqadiki xos funksiyalar uchun quydagi o‘rinli

$$(sign(x)\varphi_i^{+}, \varphi_j^{-}) = 0, \forall i, j;$$

$$(sign(x)\varphi_i^{\pm}, \varphi_j^{\pm}) = \pm\delta_{ij},$$

$$\delta_{ij} = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}$$

$P^{\pm}$ bilan quydagicha aniqlangan spektral proektorlarni belgilaymiz

$$P^{\pm}\varphi = \sum_{i=1}^{\infty} (sign(x)\varphi, \varphi_i^{\pm})\varphi_i^{\pm}$$

U holda

$$(P^{+} - P^{-})\varphi = \varphi, \qquad (\mathrm{sign}(x)(P^{+} - P^{-})\varphi, \varphi) = \|\varphi\|_0^2,$$

$$(\mathrm{sign}(x)P^{\pm}\varphi, \psi) = (\mathrm{sign}(x)\varphi, P^{\pm}\psi), \ \ \forall\varphi, \psi \in H_0 = L_2(-1,1),$$

$$\|\varphi\|_0^2 = \sum_{k=1}^{\infty} \{|sign(x)\varphi, \varphi_i^{+}|^2 + |sign(x)\varphi, \varphi_i^{-}|^2\}.$$

Quydagicha aniqlangan

$$B_n f(x) = \sum_{k=1}^{n} (u_k^{+}\varphi_k^{+} + u_k^{-}\varphi_k^{-}). \quad (4)$$

butun sonli paramtrga bog’liq bo‘lgan B_n operatorlar oilsini qaraymiz. Agar u(x,t) va f(x) funksiyalarni $L_2(-1,1)$ Gilbert fazosining elementlari sifatida qarasak , B_n

operatorlar oilasi bizning boshalang'ich masalamizga nisbatan regulyarlashtiruvchi oila bo'ladi.

Xaqiqatan bu $f^{\pm}_k$ koeffitsentlarining va chekli yig'indi uzluksizligidan B_n ning uzluksizligi kelib chiqadi.$B_n f(x)$ ketma-ketlikning u(x,t) yaqinlashishi yechimning (2) ko'rinishidan va berilgan qatorning $H_0 = L_2(-1,1)$ fazoda yaqinlashishdan kelib chiqadi.

Endi taqribiy berilganlar bo'yicha taqribiy yechimning effektivlik bahosini topamiz. Bunda qaralyotgan boshalang'ich masala Tixinov bo'yicha korrekt bo'lsin va korrektlik to'plamini quyidagicha aniqlaymiz.

$$M = \{u: \|u(x,T)\|_0 \le M\}. \quad (5)$$

Faraz qilaylik

$$\|f - f_\varepsilon\|_0 \le \varepsilon \quad (6)$$

bo'lsin.

U holda

$\|B_n\| = exp(\lambda_n^{\pm} t)$ *(2),(4),(5) shunday bo'ladi.*

$$\|B_n f(x) - u(x,t)\|_0^2 = \sum_{k=n+1}^{\infty} (exp(2\lambda_k^{+} t) f_k^{+^2} + exp(2\lambda_k^{-} t) f_k^{-^2}) = \phi_{1n}(t) + \phi_{2n}(t),$$

$$\sum_{k=1}^{\infty} (exp(2\lambda_k^{+} t) f_k^{+^2} + exp(2\lambda_k^{-} T) f_k^{-^2} \le M^2$$

kelib chiqadi. $\varphi_{1n}(t)$ funksiyani $\sum_{k=1}^{\infty}(\exp(2\lambda_k^{+}T)f_k^{+^2} \le M^2)$ shakldа baholaymiz. Agar f_k^{+} koeffitsentlar quyidagicha bo'lsa,

$f_k^{+}=0,\ k\neq n+1,$

$f^{+}_{n+1}=exp(-\lambda^{+}_{n+1}\,T)M.$

u holda $\varphi_{1n}(t)$ oxirgi shart ostida o'zining maksimal qiymatiga erishadi. Shunday qilib,

$$\varphi_{1n}(t) \le \exp(-2(T-t)\lambda^{+}_{n+1})M^2.$$

$\varphi_{2n}(t)$ uchun esa

$$\varphi_{2n}(t) = \sum_{k=n+1}^{\infty} (exp(2\lambda_k^{-} t) f_k^{-^2} \le \sum_{k=n+1}^{\infty} f_k^{-^2} = v^2(N).$$

baho o‘rinli bo‘ladi, bu yerda f uchun mos qatrning yaqinlashishi N→0 da va v→0 kelib chiqadi. $\varphi_{1n}(t)$ va $\varphi_{2n}(t)$ uchun jamlab va (6) ni e’tiborga olib topamiz.

$$\|u(x,t) - B_n f_\varepsilon(x)\|_0 \leq \|u(x,t) - B_n f(x)\|_0 + \|B_n(x) - B_n f_\varepsilon(x)\|_0 \leq e^{\lambda_n^{+t}}\varepsilon + Mexp(-\lambda_{n+1}^{+}(T-t)) + \mathrm{v}(\mathrm{n}).$$

Foydalanilgan adabiyotlar

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